On Magnetic Catalysis and Gauge Symmetry Breaking

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Non-perturbative effects of constant magnetic fields in a Higgs-Yukawa gauge model are studied using the extremum equations of the effective action for composite operators. It is found that the magnetic field induces a Higgs condensate, a fermion-antifermion condensate, and a fermion dynamical mass, hence breaking the discrete chiral symmetry of the theory. The results imply that for a non-simple group extension of the present model, the external magnetic field would induce gauge symmetry breaking. Possible cosmological applications of these results in the electroweak phase transition are suggested.

Symmetry behavior in quantum field theories under the influence of external fields has long been a topic of intensive study in theoretical physics [1]. In the present paper we are interested in particular in non-perturbative effects produced by external magnetic fields in gauge theories with scalars. Our main claim is that for gauge theories with a non-simple group of gauge symmetry and with scalar-scalar and scalar-fermion interactions, the magnetic field reinforces gauge symmetry breaking.

The observation of large-scale galactic magnetic fields in a number of galaxies, in galactic halos, and in clusters of galaxies [2] has recently stimulated a large number of works trying to explain the physical mechanism responsible for the origin of these fields. Many of the proposed generating mechanisms have compelling arguments in favor of the existence of strong primordial magnetic fields (for a review of cosmological generating mechanisms see [3] and references therein). Since primordial magnetic fields could play a significant role in particle cosmology, the investigations on the theme have recently boomed. In this context, the implications of a magnetic-field-driven gauge symmetry breaking mechanism may be important.

Several years before this renewed interest in cosmological magnetic fields, Ambøjrn and Olesen [4] considered the electroweak model in the presence of a constant magnetic field. Assuming certain special values of the couplings, they obtained a W- and Z-condensate solution forming a lattice of abelian vortex lines for a range of magnetic fields lying between $\frac{m_W^2}{e}$ and $\frac{m_W^2 \cos^2\theta}{e}$. At even larger values of the magnetic fields they found that the phase transition to a symmetric phase can be reached at temperatures lower than the critical one at zero field. This result realizes, although due to a totally different reason, an old suggestion [5] that large magnetic fields could induce the transition from the broken to the unbroken phase in the electroweak system.

More recently, the ground state of the electroweak theory in the presence of a hypermagnetic field has been investigated using either numerical or perturbative calculations [6]- [10]. The main motivation of these papers was to study the possibility that a hypermagnetic field could allow the realization of baryogenesis within the Standard Model [6]. Even though the original results [6] for the upper bound of the Higgs mass needed to have baryogenesis in the SM were quite optimist, it was quickly realized that higher loop effects [7], [10] and numerical non-perturbative calculations [8] would significantly weaken the transition. Moreover, posterior studies on which certain subtleties of the theory- like the magnetic dipole moment of the sphaleron [9] or ring diagrams contributions to the high-temperature effective potential [10]- were taken into account, concluded that albeit the hypermagnetic field strengthen the first order character of the phase transition, it is not enough to satisfy the SM baryogenesis condition [11]

$$\frac{v\left(T_c\right)}{T_c} \geq 1, \tag{1}$$

with $v\left(T_c\right)$ the Higgs vacuum expectation value (vev) at the critical temperature T_c of the electroweak phase transition. None of these studies observed the Ambøjrn and Olesen phase, nevertheless.

When a non-perturbative analytic approach is used to study field theories in external magnetic fields, new non-trivial effects are found. An important example of these non-perturbative effects is the formation of a chiral symmetry

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breaking fermion condensate $< \overline{\psi}\psi >$ and of a dynamically generated fermion mass in the presence of an external magnetic field, known in the literature as magnetic catalysis [12]. This phenomenon, which has proven to be rather universal and model independent, has recently attracted a lot of attention [13]- [18].

On normal circumstances massless fermions can condensate and acquire a dynamical mass, but the condensate appears only for sufficiently strong coupling between fermions. The new feature when a magnetic field is present is that it favors (catalyzes) the symmetry breaking by reducing to the weakest attractive coupling the strength of the interaction needed to break the symmetry. The essence of this effect is that the fermions in the lowest Landau level (LLL) constitute the effective fermionic degrees of freedom whose dynamics dominates the long wavelength behavior of the system. The phenomenon is driven by the fact that massless fermions acquire an energy gap in the presence of a magnetic field, but there is no energy gap between the vacuum and the LLL fermions. Then, in the infrared region, the dynamics of the LLL fermions dominates the fermion propagator, making it essentially D-2 dimensional. This dimensional reduction strengthen the fermion pairing dynamics [12], [19] giving rise to a fermion condensate.

It is worth to mention that the phenomenon of magnetic catalysis is not only interesting from a purely fundamental point of view, but it has potential application in condensed matter [20]- [23] and cosmology [16]. For instance, it has been recently speculated that the generation of mass through magnetic catalysis in lower dimensional models [21], [22], or in four-dimensional models with boundaries [23], could be behind the physical mechanism explaining the observed scaling of the thermal conductivity in superconducting cuprates with an externally applied magnetic field [24]. On the other hand, the magnetic catalysis could influence the character of the electroweak phase transition as suggested by the results of ref. [16].

In the present paper we consider a simple model field theory with the aim of investigating in a self-consistent way how scalar-scalar and fermion-scalar interactions in the presence of an external magnetic field can influence the stability of the vacuum. It is not intended as a realistic theory, but rather as an example of a large class of theories with scalar fields, on which dynamical symmetry breaking (either chiral or gauge) can be catalyzed by an external magnetic field. In this sense, it could be useful for condensed matter, as well as for cosmological applications. If this toy model is extended to include a non-simple gauge group theory, as it is the case of the electroweak model, the results of this paper could provide a scenario, on which, in contrast to the effect found by Ambøjrn and Olesen [4], an external magnetic field could induce gauge symmetry breaking through non-perturbative effects.

Then, let us consider the following theory of gauge, fermionic and real scalar fields described by the Higgs-Yukawa Lagrangian density

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi + g\overline{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi - \frac{\lambda}{4!}\varphi^4 - \frac{\mu^2}{2}\varphi^2 - \lambda_y\varphi\overline{\psi}\psi \tag{2}$$

It has a U(1) gauge symmetry,

$$\begin{aligned} A_{\mu} &\rightarrow A_{\mu} + \frac{1}{g}\partial_{\mu}\alpha(x) \\ \psi &\rightarrow e^{i\alpha(x)}\psi, \end{aligned} \tag{3}$$

a fermion number global symmetry

$$\psi \rightarrow e^{i\theta}\psi, \tag{4}$$

and a discrete chiral symmetry

$$\psi \rightarrow \gamma_5\psi, \qquad \overline{\psi} \rightarrow -\overline{\psi}\gamma_5, \qquad \varphi \rightarrow -\varphi \tag{5}$$

Note that a fermion mass term $m\overline{\psi}\psi$ is forbidden, since it is invariant under (3) and (4), but not under the discrete chiral symmetry (5).

To study the vacuum instabilities that could arise in the theory (2) under the influence of an external constant magnetic field B, we need to solve the extremum equations of the effective action Γ for composite operators [25], [26]

$$\frac{\delta\Gamma(\varphi_c, \overline{G})}{\delta\overline{G}} = 0, \tag{6}$$

$$\frac{\delta\Gamma(\varphi_c, \overline{G})}{\delta\varphi_c} = 0 \tag{7}$$

where $\overline{G}(x,x) = \sigma(x) = \langle 0 \mid \overline{\psi}(x)\psi(x) \mid 0 \rangle$ is a composite fermion-antifermion field and φ_c represents the vev of the Higgs field. Thanks to the discrete chiral symmetry (5), it is enough to consider only one composite field. We choose the composite field $\overline{G}(x,x)$, ignoring the second possible one, $\pi(x) = \langle 0 \mid \overline{\psi}(x) i\gamma_5 \psi(x) \mid 0 \rangle$, since the effective action can be a function only of the chirally invariant combination $\rho^2 = \sigma^2 + \pi^2$.

The loop expansion of the effective action Γ for composite operators [25], [26] can be expressed as

$$\Gamma\left(\overline{G},\varphi_c\right) = S\left(\varphi_c\right) - iTr\ln\overline{G}^{-1} + i\frac{1}{2}Tr\ln D^{-1} + i\frac{1}{2}Tr\ln\Delta^{-1} - iTr\left[G^{-1}\left(\varphi_c\right)\overline{G}\right] + \Gamma_2\left(\overline{G},\varphi_c\right) + C \tag{8}$$

In Eq. (8) C is a constant and $S\left(\varphi_c\right)$ is the classical action evaluated in the scalar vev (Higgs condensate) φ_c. The bar on the fermion propagator $\overline{G}\left(x,y\right)$ means that it is taken full, while the non-bar notation indicates free propagators, as it is the case for the gauge propagator $D_{\mu\nu}(x-y) = \int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-x')}}{q^2-i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2-i\epsilon}\right)$, and the scalar one $\Delta(x-y) = \int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-x')}}{q^2+M^2-i\epsilon}$, with $M^2 = \frac{\lambda}{2}\varphi_c^2 + \mu^2$. In general $\Gamma_2\left(\overline{G},\varphi_c\right)$ represents the sum of two and higher loop two-particle irreducible vacuum diagrams. In the current approximation, as all propagators but the fermion's are taken free, Γ_2 is two-particle irreducible with respect to fermion lines only [26]. In the present weakly coupled theory one can use the lowest (two-loop) approximation for Γ_2. This corresponds to the so called quenched ladder approximation, on which all vertices are taken bare. In this case Γ_2 is

$$\begin{aligned}
\Gamma_2\left(\overline{G},\varphi_c\right) = {} & \frac{g^2}{2}\int d^4x d^4y tr\left[\overline{G}\left(x,y\right)\gamma^\mu\overline{G}\left(y,x\right)\gamma^\nu D_{\mu\nu}(x,y)\right] \\
& -\frac{g^2}{2}\int d^4x d^4y tr\left(\gamma^\mu\overline{G}\left(x,x\right)\right) D_{\mu\nu}(x-y) tr\left(\gamma^\nu\overline{G}\left(y,y\right)\right) \\
& +\frac{\lambda_y^2}{2}\int d^4x d^4y tr\left[\overline{G}\left(x,y\right)\overline{G}\left(y,x\right)\Delta(x,y)\right] \\
& -\frac{\lambda_y^2}{2}\int d^4x d^4y tr\left(\overline{G}\left(x,x\right)\right)\Delta(x-y) tr\left(\overline{G}\left(y,y\right)\right)
\end{aligned} \tag{9}$$

The extremum equations (6) and (7) correspond, respectively, to the Schwinger-Dyson (SD) equation for the fermion mass operator Σ (gap equation), and to the usual minimum equation for the expectation value of the scalar field, which in the presence of the magnetic field has to be determined in a self-consistent way, that is, simultaneously with the gap equation.

Although we have introduced a bare scalar mass μ in (2), because we are interested in the possibility of a dynamically generated scalar mass, we take the limit $\mu \to 0$ at the end of our calculations.

The second to fourth terms in the effective action (8) correspond to the one-loop contribution. Their evaluation is quite straightforward (the scalar self-interaction can be renormalized in the usual way [27]), with the exception perhaps of the fermion contributions, which contain the background magnetic field. Then, let us calculate explicitly the one-loop fermion contribution coming from the term

$$\Gamma_f^{(1)} = -iTr\left[G^{-1}\left(\varphi_c\right)\overline{G}\right] \tag{10}$$

in (8). Here $G^{-1}\left(\varphi_c\right)$ is the free fermion inverse propagator in the presence of a constant magnetic field B along the third axis,

$$G^{-1}\left(x,y,\varphi_c\right) = \sum_k \int \frac{dp_0 dp_2 dp_3}{\left(2\pi\right)^4} E_p\left(x\right)\left(\gamma.\overline{p} + m_0\right)\overline{E}_p\left(y\right), \tag{11}$$

with $\overline{p} = (p_0, 0, -\sqrt{2gBk}, p_3)$, $m_0 = \lambda_y\varphi_c$ the fermion mass appearing after the shift $\varphi \to \varphi + \varphi_c$ in the Higgs field, and $\overline{G}\left(x,y\right)$ the full fermion propagator, which can be written as [13]

$$\overline{G}\left(x,y\right) = \sum_k \int \frac{dp_0 dp_2 dp_3}{\left(2\pi\right)^4} E_p\left(x\right)\left(\frac{1}{\gamma.\overline{p} + \Sigma(p)}\right)\overline{E}_p\left(y\right) \tag{12}$$

In the above equations we have introduced the Ritus' E_p functions [28]. These orthonormal function-matrices provide an alternative method to the Schwinger's approach to problems of QFT on electromagnetic backgrounds[1].

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[1]For an application of Ritus' method to the QED Schwinger-Dyson equation in a magnetic field see the second paper of ref. [13].

The E_p representation is obtained forming the eigenfunction-matrices of the fermion mass operator

$$E_p(x) = \sum_\sigma E_{p\sigma}(x)\Delta(\sigma), \tag{13}$$

where

$$\Delta(\sigma) = diag(\delta_{\sigma 1}, \delta_{\sigma -1}, \delta_{\sigma 1}, \delta_{\sigma -1}), \qquad \sigma = \pm 1, \tag{14}$$

and the $E_{p\sigma\chi}$ functions are given by

$$E_{p\sigma}(x) = N(n) e^{i(p_0 x^0 + p_2 x^2 + p_3 x^3)} D_n(\rho) \tag{15}$$

with $D_n(\rho)$ being the parabolic cylinder functions [29] of argument $\rho = \sqrt{2gB}(x_1 - \frac{p_2}{gB})$ and positive integer index

$$n = n(k,\sigma) \equiv k + \frac{\sigma}{2} - \frac{1}{2} \qquad n = 0, 1, 2, ..., \tag{16}$$

and $N(n) = (4\pi gB)^{\frac{1}{4}}/\sqrt{n!}$ being a normalization factor. Here p represents the set $(p_0, p_{2,} p_3, k)$, which determines the eigenvalue $\overline{p}^2 = -p_0^2 + p_3^2 + 2gBk$ in $(\gamma^\mu\,(i\partial_\mu - gA_\mu))^2\psi_p = \overline{p}^2\psi_p$ (for details and notation see [13] and [16]). In Eq. (15) we are considering the case of a purely magnetic field background (crossed field case) directed along the z-direction (without loss of generality we assume that $sign(gB) = 1$).

One can easily check that the E_p functions are orthonormal

$$\int d^4x \overline{E}_{p'}(x) E_p(x) = (2\pi)^4 \widehat{\delta}^{(4)}(p - p') \equiv (2\pi)^4 \delta_{kk'} \delta(p_0 - p_0') \delta(p_2 - p_2') \delta(p_3 - p_3') \tag{17}$$

and complete

$$\sum_k \int d^3p E_p(x) \overline{E}_p(y) = \sum_k \int dp_0 dp_2 dp_3 E_p(x) \overline{E}_p(y) = (2\pi)^4 \delta^{(4)}(x - y) \tag{18}$$

Here we have used $\overline{E}_p(x) = \gamma^0 E_p^\dagger \gamma^0$.

Using Eqs.(11) and (12) in (10), the last one can be expressed as

$$\Gamma_f^{(1)} = -i \int d^4x d^4y \sum_k \int \frac{d^3p}{(2\pi)^4} \sum_{k'} \int \frac{d^3p'}{(2\pi)^4} Tr\{E_p\,(x)\,(\gamma.\overline{p} + m_0)\,\overline{E}_p\,(y)$$

$$\times E_{p'}\,(y) \left(\frac{1}{\gamma.\overline{p}' + \Sigma(p')}\right) \overline{E}_{p'}\,(x)\} \tag{19}$$

Making use of the property (17) one can easily integrate in y and p' to obtain

$$\Gamma_f^{(1)} = -i \int d^4x \sum_k \int \frac{d^3p}{(2\pi)^4} Tr \left\{ E_p\,(x) \left(\frac{\gamma.\overline{p} + m_0}{\gamma.\overline{p} + \Sigma(p)} \right) \overline{E}_p\,(x) \right\} \tag{20}$$

At this point we need to consider the structure of the mass operator Σ introduced in ref. [17]

$$\sum(\overline{p}) = Z_\parallel(\overline{p}) \gamma \cdot \overline{p}_\parallel + Z_\perp(\overline{p}) \gamma \cdot \overline{p}_\perp + m(\overline{p}) \tag{21}$$

where $\overline{p}_\parallel = (p_0, 0, 0, p_3)$, $\overline{p}_\perp = (0, 0, -\sqrt{2gBk}, 0)$, and $m(\overline{p})$ is the total dynamical fermion mass that in principle depends on the momentum $\overline{p}$. Then, taking into account (21), the contribution (20) can be written as

$$\Gamma_f^{(1)} = -i \int d^4x \sum_k \int \frac{d^3p}{(2\pi)^4} Tr \left\{ E_p\,(x) \left(\frac{\gamma.\overline{p} + m_0}{(1 + Z_\parallel)\gamma.\overline{p}_\parallel + (1 + Z_\perp)\gamma \cdot \overline{p}_\perp + m(\overline{p})} \right) \overline{E}_p\,(x) \right\} \tag{22}$$

Using the trace properties and the orthonormality of the E_p, the integral in x yields

$$\Gamma_f^{(1)} = -i\,(2\pi)^4\,\delta^{(3)}(0)\sum_k\int\frac{d^3p}{(2\pi)^4}Tr\left\{\frac{\gamma.\overline{p}+m_0}{(1+Z_{\|})\gamma.\overline{p}+(1+Z_\perp)\gamma\cdot\overline{p}_\perp+m(\overline{p})}\right\} \tag{23}$$

where the notation $\delta^{(3)}(k)=\delta(k_0)\delta(k_2)\delta(k_3)$ is understood. After taking the trace, integrating in p_2 and doing the Wick rotation to Euclidean coordinates, we obtain

$$\Gamma_f^{(1)} = 8\pi gB\delta^{(4)}(0)\sum_k\int dp_4dp_3\frac{(1+Z_{\|})\overline{p}_{\|}^2+(1+Z_\perp)\overline{p}_\perp^2+m(\overline{p})m_0}{(1+Z_{\|})^2\overline{p}_{\|}^2+(1+Z_\perp)^2\overline{p}_\perp^2+m^2(\overline{p})} \tag{24}$$

The two-loop contributions are a little more involved. Since we have not enough space in a letter to give all the detailed calculations, we will explicitly show, for the sake of understanding, the evaluation of one term. The others can be found in a similar way. The complete calculation will be published elsewhere.

First, notice that the second and fourth term in Eq.(9) generate tadpole diagrams in the SD equation (6). It is easy to realize that the tadpole diagram from the gauge-fermion vertex vanishes. However, the tadpole associated to the scalar-fermion vertex is not zero when $\varphi_c\neq 0$ and in this case it has a significant contribution to the gap equation, as shown below. Let us evaluate this tadpole contribution, which we denote by $\sum^T$.

$$\sum\nolimits^T(x,y)=i\frac{\delta\Gamma_2^T}{\delta G}=-i\lambda_y^2\delta^4(x-y)\int d^4z\Delta(x-z)tr\left[\overline{G(z,z)}\right] \tag{25}$$

We can transform Eq.(25) to momentum space with the help of the $E_p\,(x)$ functions [13], [16] to obtain

$$\int d^4xd^4y\overline{E}_p\,(x)\sum\nolimits^T(x,y)E_{p'}\,(y)=(2\pi)^4\widehat{\delta}^{(4)}(p-p')\sum\nolimits^T(\overline{p})$$

$$\begin{aligned}&=-i\lambda_y^2\int d^4xd^4z\overline{E}_p\,(x)\int\frac{d^4q}{(2\pi)^4}\frac{e^{iq(x-z)}}{q^2+M^2+i\in}\\&\times\sum_{k"}\int\frac{d^3p"}{(2\pi)^4}Tr\left\{E_{p"}\,(z)\left(\frac{1}{\gamma.\overline{p}"+\sum(\overline{p}")}\right)\overline{E}_{p"}\,(z)\right\}E_{p'}\,(x)\end{aligned} \tag{26}$$

Using the properties of the trace and taking into account that [13]

$$\begin{aligned}\int d^4xe^{iqx}\overline{E}_p\,(x)\,E_{p'}\,(x)\;&=(2\pi)^4\,\delta^{(3)}(p'+q-p)e^{iq_1(p_2'+p_2)/2gB}e^{-\widehat{q}_\perp^2/2}\\&\times\sum_{\sigma\sigma'}\frac{e^{i(n-n')\varphi}}{\sqrt{n(k,\sigma)!n'(k',\sigma')!}}J_{nn'}(\widehat{q}_\perp)\Delta(\sigma)\delta_{\sigma\sigma'}\end{aligned} \tag{27}$$

one can integrate in x and z to find

$$\begin{aligned}(2\pi)^4\widehat{\delta}^{(4)}(p-p')\sum\nolimits^T(\overline{p})&=-i\lambda_y^2\int d^4q\sum_{k"}\int d^3p"\,\delta^{(3)}(q)\delta^{(3)}(p'+q-p)\\&\times e^{-\widehat{q}_\perp^2}\frac{e^{iq_1(p_2'+p_2-2p_2")/2gB}}{q^2+M^2+i\in}\sum_\sigma\frac{e^{i(n-n')\varphi}}{\sqrt{n(k,\sigma)!n'(k',\sigma)!}}J_{nn'}(\widehat{q}_\perp)\Delta(\sigma)\\&\times\sum_{\sigma"}\left\{Tr\left[\Delta\left(\sigma"\right)\frac{1}{\gamma.\overline{p}"+\sum(\overline{p}")}\right]\frac{1}{n"(k",\sigma")!}J_{n"n"}(\widehat{q}_\perp)\right\},\end{aligned} \tag{28}$$

This equation can be further simplified after taking the trace and using the small $\widehat{q}_\perp^2$ approximation of the $J-$functions

$$J_{nn"}(\widehat{q}_\perp)\rightarrow\frac{[\max(n,n")]!}{|n-n"|!}\left[i\widehat{q}_\perp\right]^{|n-n"|}\rightarrow n!\delta_{nn"}, \tag{29}$$

which can be justified by the presence of the exponential factor $e^{-\widehat{q}_\perp^2}$ in the integrand of Eq.(28). Moreover, thanks to the delta $\delta^{(3)}(q)$, the integrations in q_0, q_2, q_3 are trivial. Thus, taking into account all these and using the properties of the Δ matrices [13], we arrive at

$$(2\pi)^4\widehat{\delta}^{(4)}(p-p')\sum\nolimits^T(\overline{p}) = -2i\lambda_y^2\delta^{(4)}(p'-p)\int dq_1 \sum_{k"}\int d^3p" e^{-\widehat{q}_\perp^2}\frac{e^{iq_1(p_2'+p_2-2p_2")/2gB}}{q_1^2+M^2+i\in}$$

$$\times\left\{\frac{2m(\overline{p}")}{\left(1+Z_{_\parallel}\right)^2\overline{p}_{_\parallel}"^2+(1+Z_\perp)^2\overline{p}_{_\perp}"^2+m^2(\overline{p}")}\right\} \tag{30}$$

Finally, after integrating in q_1 and p_2 and transforming to Euclidean space, we get

$$\sum\nolimits^T(\overline{p}) = \frac{\lambda_y^2}{2\pi^3}\frac{gB}{M^2}\sum_{k"}\int dp_4"dp_3"\frac{m(\overline{p}")}{\left(1+Z_{_\parallel}\right)^2\overline{p}_{_\parallel}"^2+(1+Z_\perp)^2\overline{p}_{_\perp}"^2+m^2(\overline{p}")} \tag{31}$$

It can be shown [2] that $Z_{_\parallel}=Z_\perp=0$ is a solution of Eqs.(6) and (7). In general, the dynamical mass depends on the momentum, but it is reasonable to expect [12], [18] that a constant solution exists in the infrared region $k << \sqrt{gB}$, which is the one of interest. Hence, we assume $m(p")\approx m(o)=m$. Thus, the tadpole contribution to the gap equation becomes

$$\sum\nolimits^T \simeq \frac{1}{\pi^2}\frac{\lambda_y^2}{\lambda\varphi_c^2}gBm\ln\left(\frac{gB}{m^2}\right) = -2\frac{\lambda_y^2}{\lambda\varphi_c^2} < \overline{\psi}\psi >, \tag{32}$$

where $< \overline{\psi}\psi >= iTr\left\{\overline{G}(x,x)\right\} = -\frac{gBm}{2\pi^2}\ln\left(\frac{gB}{m^2}\right)$ is the fermion condensate [13], [14].

Taking into account all the contributions to Eqs. (6) and (7) and keeping only the most important ones at large field B, one arrives to the following approximated minimum equations for the fermion mass and the Higgs vev respectively,

$$m \simeq m_0 + \frac{\alpha}{4\pi}m\ln^2\left(\frac{gB}{m^2}\right) + \frac{1}{\pi^2}\frac{\lambda_y^2}{\lambda\varphi_c^2}gBm\ln\left(\frac{gB}{m^2}\right) \tag{33}$$

$$\left(\frac{\lambda}{6}-\frac{25}{384\pi^2}\lambda^2\right)\varphi_c^3 - \lambda_y\frac{gB}{2\pi^2}m\ln\left(\frac{gB}{m^2}\right)\simeq 0 \tag{34}$$

They can be further simplified by noting that one can neglect the terms $\sim \lambda^2$ coming from the one-loop scalar self-interaction in Eq. (34), compared to the term coming from the fermion condensate contribution $\sim < \overline{\psi}\psi >$. Then Eq. (34) leads to

$$\varphi_c^3 \simeq \frac{\lambda_y}{\lambda}\frac{3gB}{\pi^2}m\ln\left(\frac{gB}{m^2}\right) \tag{35}$$

Using it back in Eq.(33) , it can be straightforward found that

$$m \simeq \frac{1}{\sqrt{\kappa}}\sqrt{gB} \tag{36}$$

where the coefficient κ satisfies

$$\kappa\ln\kappa \simeq 1.4\frac{\lambda}{\lambda_y^4} \tag{37}$$

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[2]The demostration that $Z_{_\parallel}=Z_\perp=0$ is a solution of the gap equation in the present theory can be done along the same line of reasoning followed in the Appendix of the first paper of ref. [16].

The corresponding solution for the Higgs vev is

$$\varphi_c \approx \frac{0.8}{\kappa^{1/2}\lambda_y}\sqrt{gB} \tag{38}$$

Note that there is no zero solution for the scalar vev in this large field approximation. Both, the minimum of the scalar field and the dynamically generated mass are driven by the external magnetic field. Since the fermion mass vev breaks the discrete chiral symmetry (5) this model might be considered as one more example of the phenomenon of magnetic catalysis. However, if the current model is extended to include complex scalars (complex scalars do not change at all the conclusions of this paper) and a non-simple gauge group, as the electroweak model, the symmetry breaking phenomenon will have a different nature. There we do not have chiral symmetry, but the magnetic-field-driven scalar vev will break the gauge symmetry by giving mass to the gauge fields coupled to it. Thus, as we claimed at the beginning of the paper, in richer models with scalar fields, the magnetic field can catalyze gauge symmetry breaking through non-perturbative effects.

Comparing the induced fermion dynamical mass Eq.(36) with the mass generated when no scalar field is present [12], [13], $m \simeq \sqrt{|gB|}\exp\left[-\sqrt{\frac{4\pi^2}{g^2}}\right]$, or with the one when the vev of the scalar field is fine-tuned to zero [16], $m \approx \sqrt{|gB|}\exp\left[-\sqrt{\frac{\pi}{\sqrt{\frac{g^2}{4\pi}+\frac{\lambda_y^2}{4\pi}}}}\right]$, one realizes that the scalar interactions, when taken into account in a self-consistent way, can dramatically strengthen the generation of mass. This observation is easy to corroborate by direct calculations of the mass (36) for typical values of the Yukawa coupling λ_y and the scalar self-coupling λ. For instance, if we take $\lambda_y = 0.7$, which would be the approximate value of the Yukawa coupling for the top quark, and $\lambda = 0.4$, which would correspond to a Higgs mass of 115 Gev, we would find $m \simeq 0.6\sqrt{|gB|}$. The same Yukawa coupling, on the other hand, would give just $m \simeq 10^{-5}\sqrt{|gB|}$ if the scalar vev is fine-tuned to zero [16]. In the case of pure gauge interactions, as for instance in QED, the generated mass would be even much smaller [12], [13].

A non-simple group extension of the model discussed in this work could be of interest as an effective theory in condensed matter problems, where $SU(2)\times U(1)$ gauge theories (without Higgs fields) have been previously proposed to describe the rich phase structure of high T_c superconductors [21].

It seems, however, that the most immediate physical extension of the present model would be the electroweak theory. This case is particularly interesting in the light of the recent works on the role of magnetic fields in electroweak baryogenesis [6]- [10]. In view of the magnetic-field-driven non-perturbative enlargement of the Higgs vev, Eq. (38), it is possible that in the electroweak theory the enlargement will be large enough to guarantee the baryogenesis condition (1) . It remains therefore as an open question whether the effect found in this paper can influence the recent conclusions [6]- [10] about baryogenesis in the presence of primordial magnetic fields.

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